Surface Reconstruction from Point Clouds without Normals by Parametrizing the Gauss Formula

Siyou Lin[1], Dong Xiao[1], Zuoqiang Shi[1,2], Bin Wang[1,3]

[1] Tsinghua University
[2] Yanqi Lake Beijing Institute of Mathematical Sciences and Applications (BIMSA)
[3] Beijing National Research Center for Information Science and Technology (BNRIST)

SIGGRAPH 2023
LOS ANGELES+ 6-10 AUG

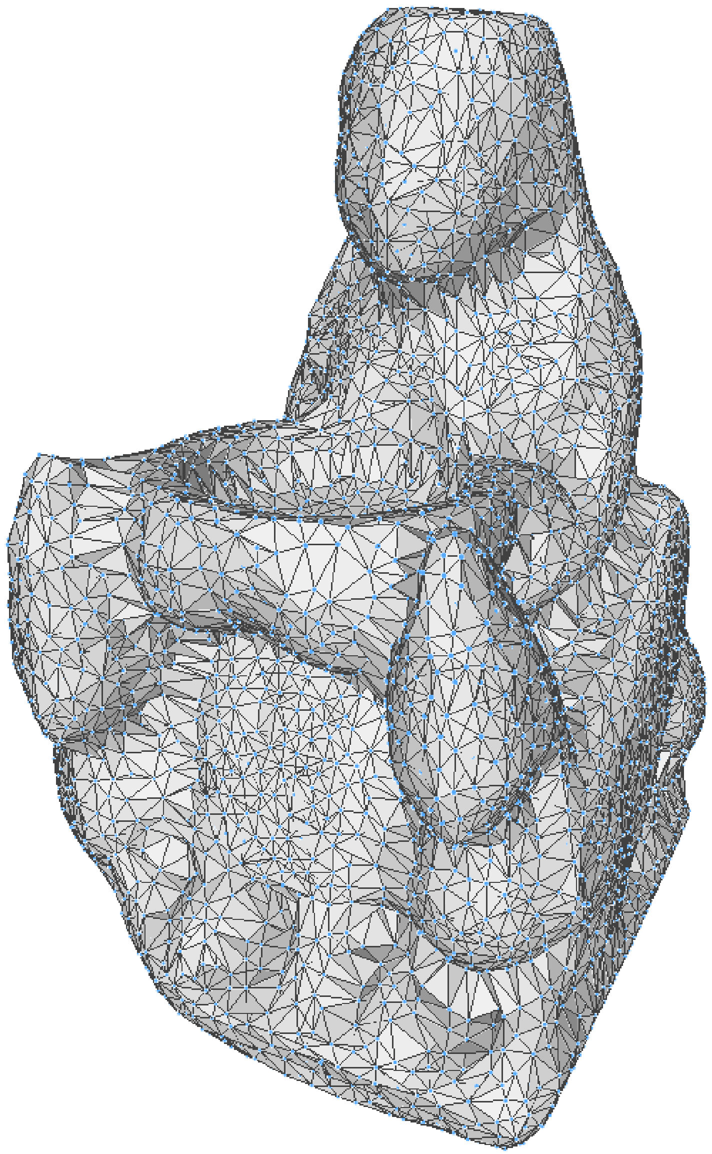
Overview

- Background and Related Work
- Motivation and Methods
- Results and Comparisons
- Conclusions, Limitations and Future Work

1.1 Existing Explicit Methods

Directly figure out how points are connected

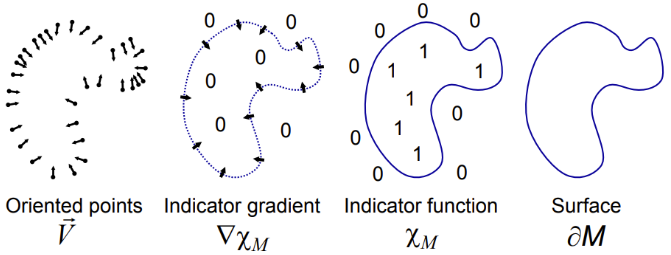
- Amenta et al. 1998, 2000, 2001
- Dey and Goswami, 2004
- α-shapes [Edelsbrunner and Mücke 1994]
- Ball pivoting [Bernardini et al. 1999]
- ...



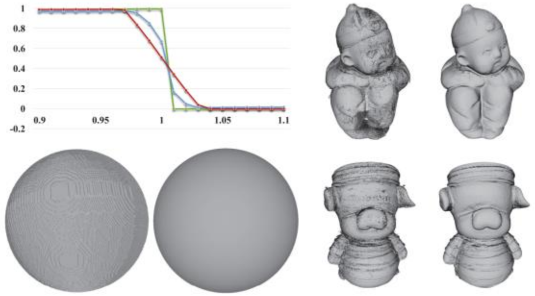
- May not guarantee watertightness
- Usually not robust to noise

1.2 Existing Implicit Methods

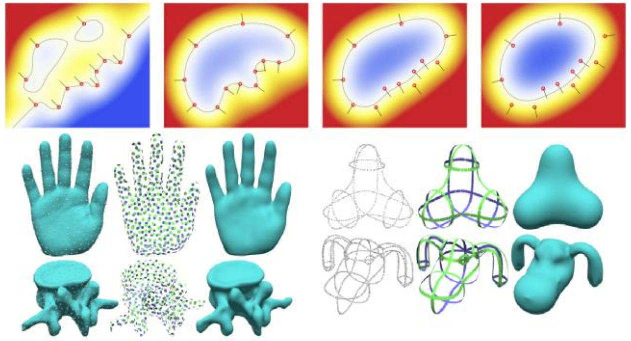
Compute implicit functions and extract level-set surface.



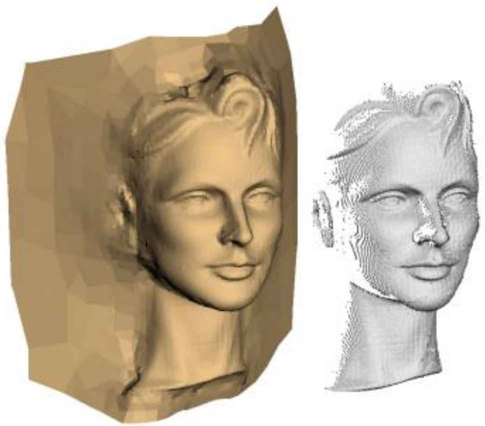
Poisson Reconstruction [Kazhdan et al. 2006]



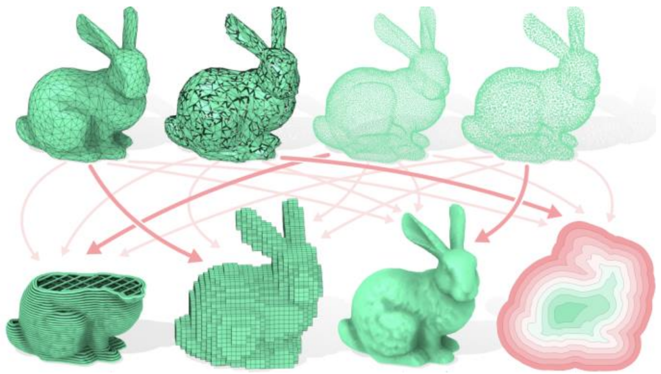
Gauss reconstruction [Lu et al. 2018]



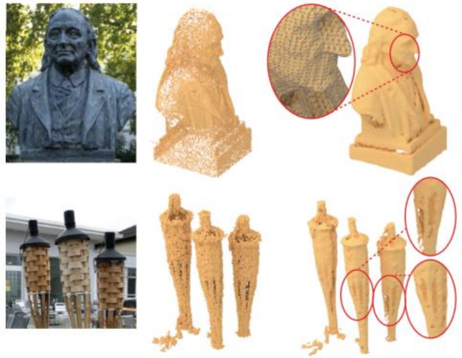
VIPSS [Huang et al. 2019]



Smooth signed distance [Calakli and Taubin 2011]



Fast winding number [Barill et al. 2018]

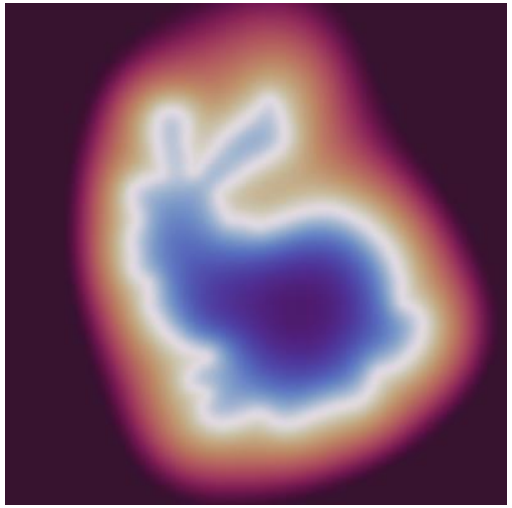


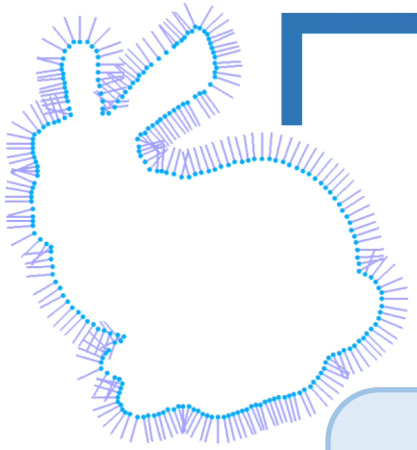
Points2Surf [Erler et al. 2020]

1.2 Existing Implicit Methods

Parametric: Poisson reconstruction [Kazhdan et al. 2006]

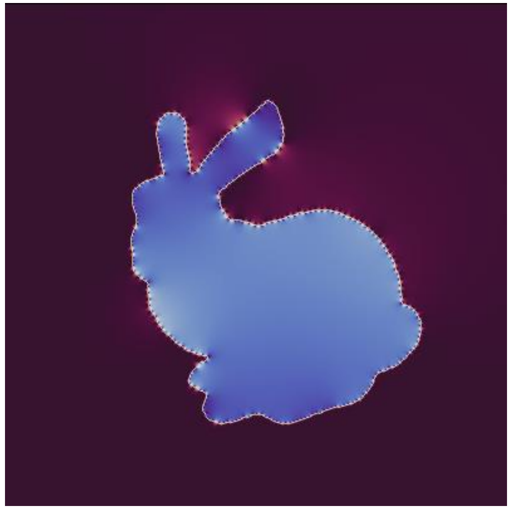
Find F in a parametric space such that $\Delta F(x) = \nabla \cdot V(x)$





Nonparametric: Winding number [Barill et al. 2018]

$$F(x) = \int_{y \text{ on surface}} \frac{-(x-y)}{4\pi|x-y|^3} \cdot V(y)dS(y) = \begin{cases} 1 & \text{for } x \text{ inside} \\ 0 & \text{for } x \text{ outside} \end{cases}$$



2 Motivation and Method

Are normals necessary?

- Normals can also be viewed as parameters.

Given $V(x)$, compute $F(x)$ based on certain rules → Each rule defines a mapping: $V \longmapsto F(x; V)$

Methods requiring normals | Normals as parameters

A parametric function space

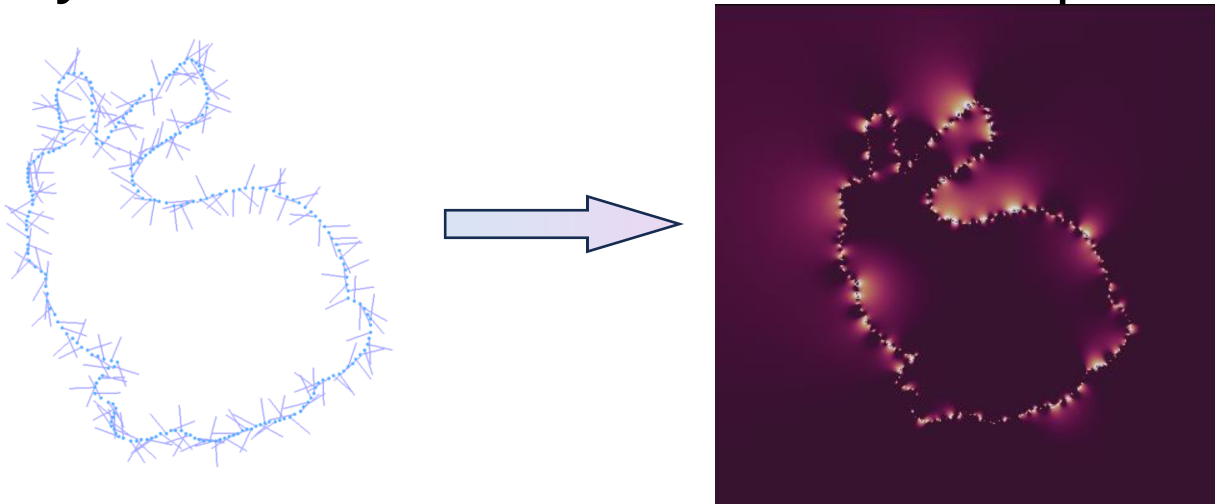
↓

Gauss formula

↑

How to define validity of an implicit function

- Arbitrary normals WON'T lead to a valid implicit function.



Tsinghua University

2.1 Parametrizing the Gauss Formula

Winding number

$$F(x) = \int_{y \text{ on surface}} \frac{-(x-y)}{4\pi|x-y|^3} \cdot V(y)dS(y) = \begin{cases} 1 & \text{for } x \text{ inside} \\ 0 & \text{for } x \text{ outside} \end{cases}$$

Gauss formula

$$F(x) = \int_{y \text{ on surface}} \frac{-(x-y)}{4\pi|x-y|^3} \cdot V(y)dS(y) = \begin{cases} 1 & \text{for } x \text{ inside} \\ 1/2 & \text{for } x \text{ on surface} \\ 0 & \text{for } x \text{ outside} \end{cases}$$

Known kernel function

Normals as unknown parameters

Constraint

We can solve for normals from this equation!

2.1 Parametrizing the Gauss Formula

Gauss formula

$$F(x) = \int_{y \text{ on surface}} \frac{-(x-y)}{4\pi|x-y|^3} \cdot V(y)\,dS(y) = \begin{cases} 1 & \text{for } x \text{ inside} \\ 1/2 & \text{for } x \text{ on surface} \\ 0 & \text{for } x \text{ outside} \end{cases}$$

Discretized

$$F(x_i) \approx \sum_j \frac{-(x_i - y_j)}{4\pi\left|x_i - y_j\right|^3} \cdot \mu_j = 1/2 \quad \text{for } x_i, y_j \text{ on surface}$$

As matrix

$$A\mu = 1/2 \qquad \text{for } A \in R^{N\times 3N}, \mu \in R^{3N}$$

Parametric Gauss Reconstruction (PGR)

2.2 Singularity Problem

We want to solve $A\mu = 1/2$ for $A_{ij} = \frac{-(x_i - y_j)}{4\pi |x_i - y_j|^3}$

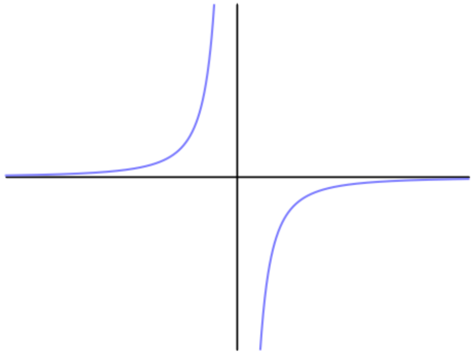
Singular at $x_i \approx y_j$

2.2 Singularity Problem

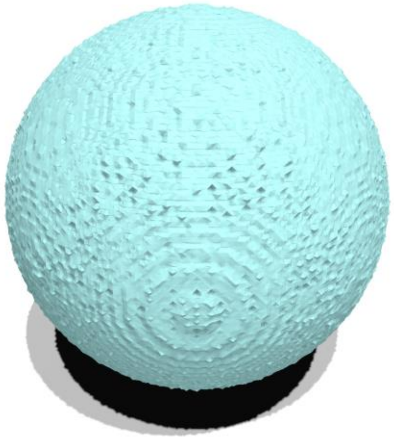
We want to solve $A\mu = 1/2$ for $A_{ij} = \frac{-(x_i - y_j)}{4\pi|x_i - y_j|^3}$

Singular at $x_i \approx y_j$

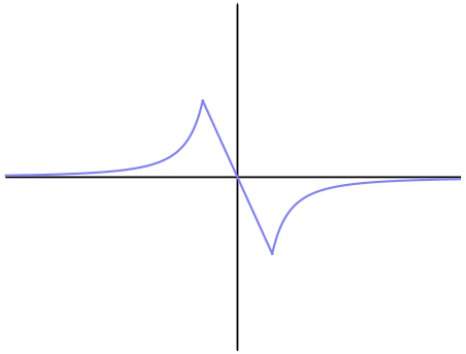
- Jagged and noisy surface if directly used in our formulation.



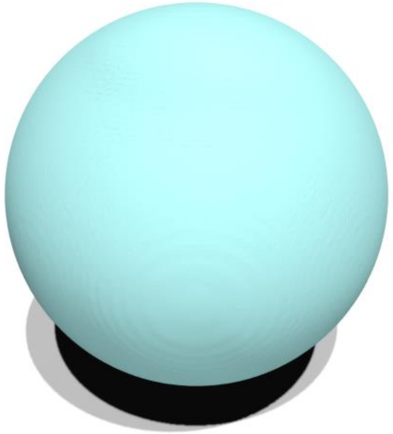
Singular kernel



GR [Lu et al. 2018]



Modified kernel

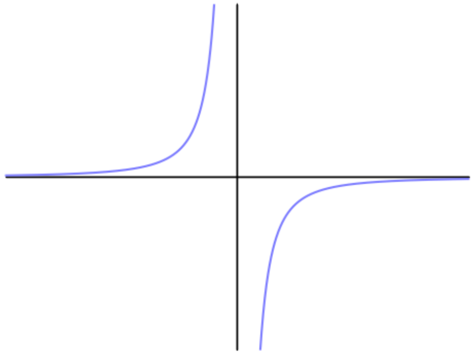


2.2 Singularity Problem

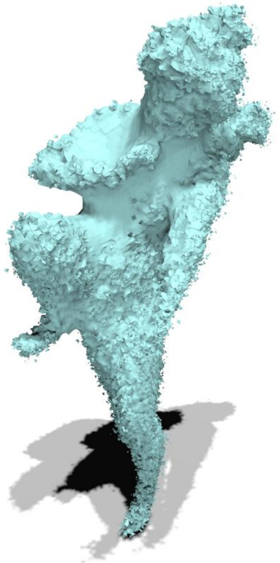
We want to solve $A\mu = 1/2$ for $A_{ij} = \frac{-(x_i - y_j)}{4\pi|x_i - y_j|^3}$

Singular at $x_i \approx y_j$

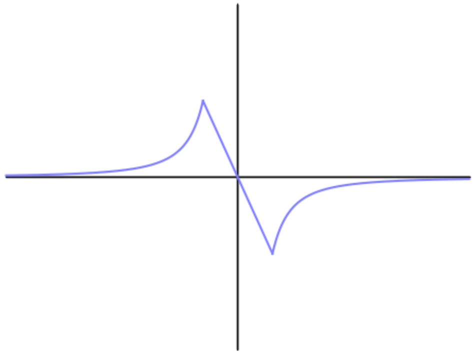
- Jagged and noisy surface if directly used in our formulation.
- Can be used for smoothing noisy inputs



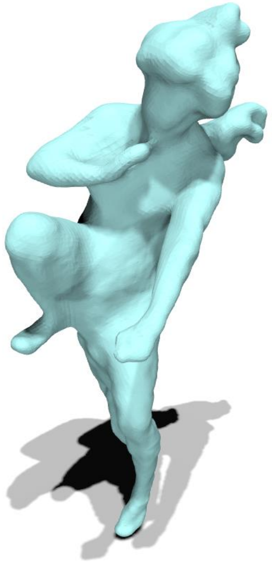
Singular kernel



GR [Lu et al. 2018]



Modified kernel



2.3 Underdetermined System

Naive PGR formulation

$A\mu = 1/2$ for $A \in R^{N\times 3N}, \mu \in R^{3N}$

Underdetermined, non-square and dense

Normal equation

$AA^T\xi = 1/2$ and $\mu = A^T\xi$ for $AA^T \in R^{N\times N}$

Ill-conditioned and dense

Adaptive regularization

$(AA^T + (\alpha - 1)\cdot diag(AA^T))\xi = 1/2$

Dense

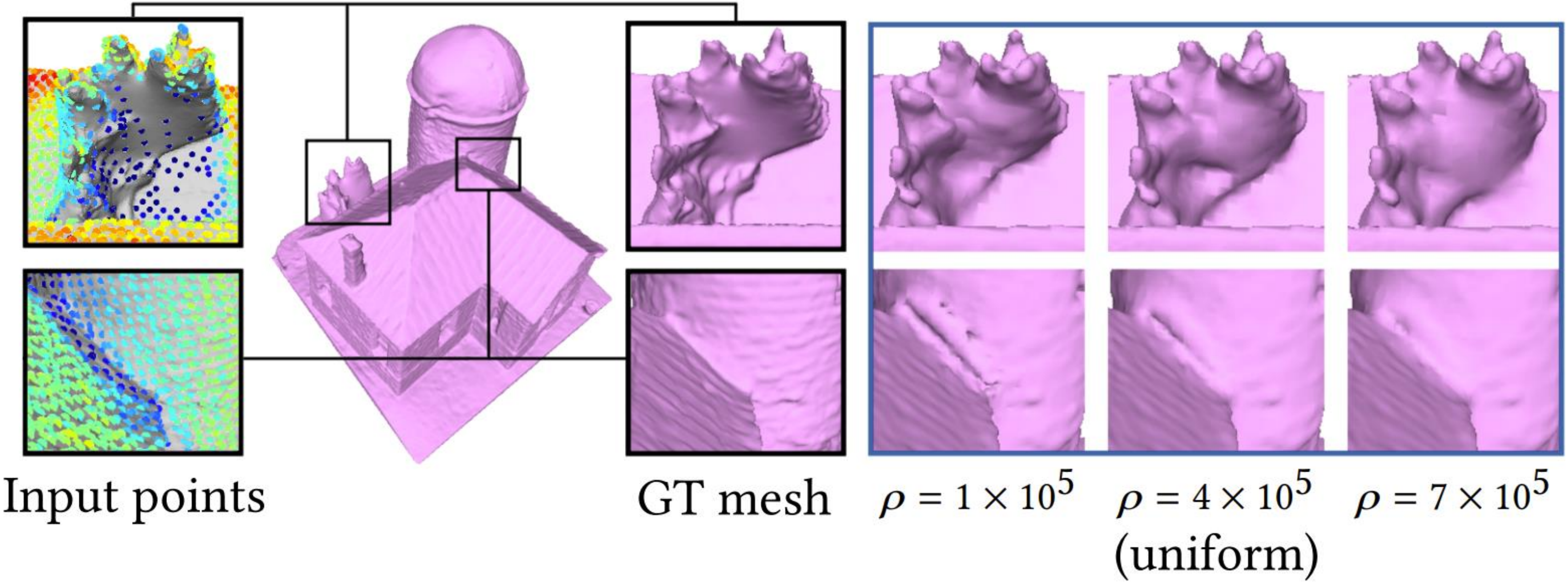
...and solve with Conjugate Gradients

2.4 Adaptive Regularization

Uniform regularization

$$(AA^T + \rho \cdot I)\xi = 1/2$$

- Difficult to choose a proper ρ
- Cannot be used for nonuniform points

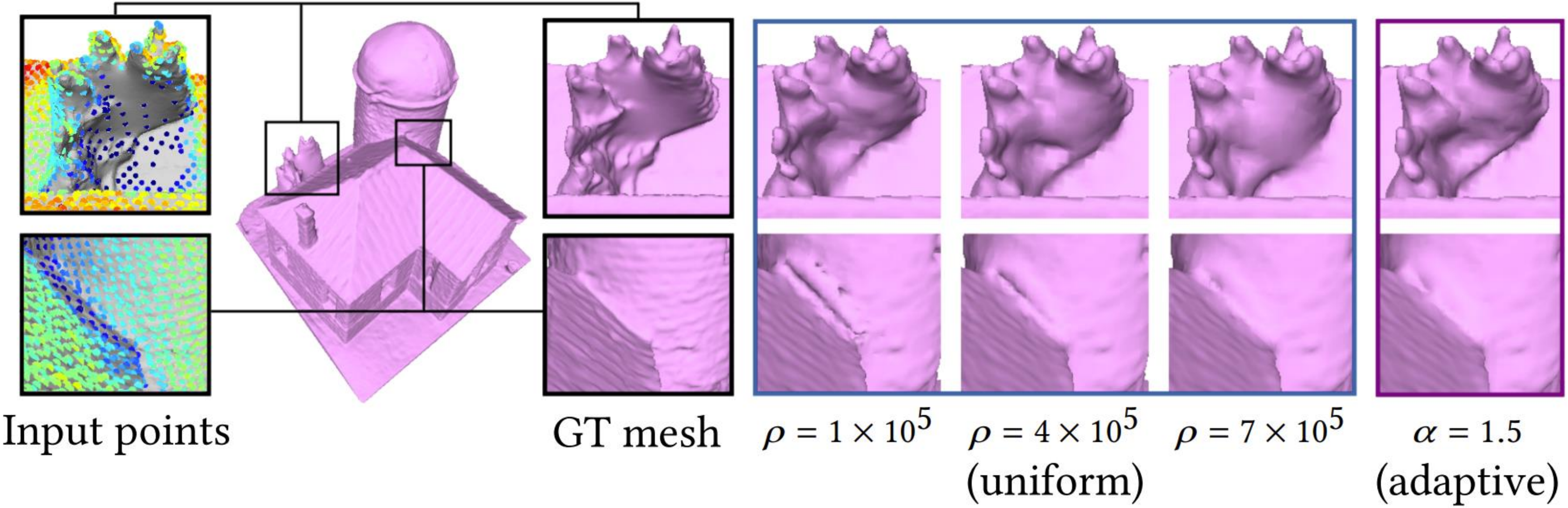


2.4 Adaptive Regularization

Adaptive regularization

$$(AA^T + (\alpha - 1) \cdot diag(AA^T))\xi = 1/2$$

- α can be chosen more easily
- Deals with nonuniform points



3 Results and Comparisons

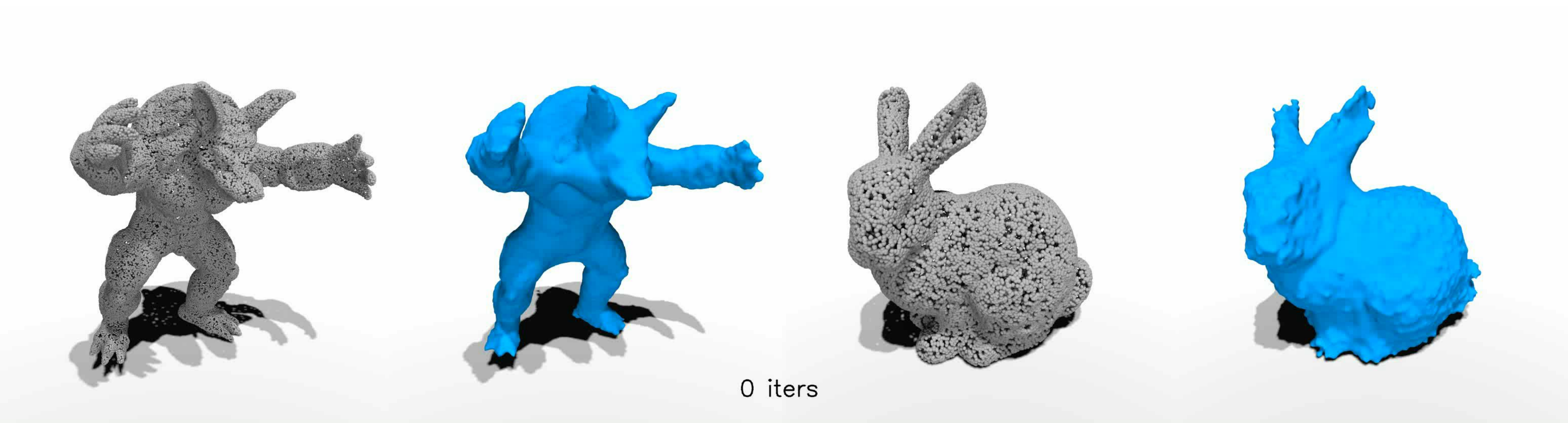
- Screened Poisson Reconstruction **(SPR)** [Kazhdan et al. 2013]: The golden standard
- Gauss Reconstruction **(GR)** [Lu et al. 2018], which also uses Gauss formula
- Points2surf **(P2S)** [Erler et al. 2020]: a learning based method
- **VIPSS** [Huang et al. 2019]: a radial-basis function method

SPR and GR are facilitated with

- **PCA** normal estimation in MeshLab
- **PCPNet** normal estimation [Guerrero et al 2018]

3.1 Convergence (Conjugate Gradients)

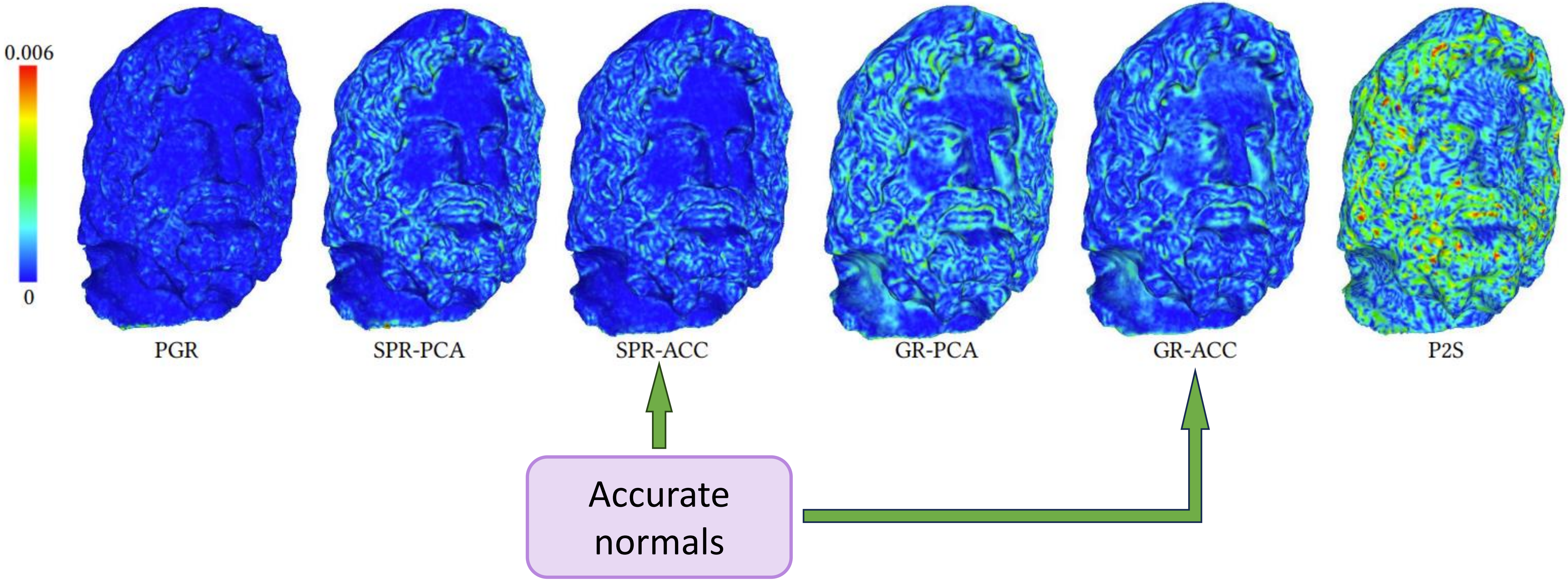
$$(AA^T + \alpha \cdot diag(AA^T))\xi = 1/2, \qquad \xi_{init} = 0$$



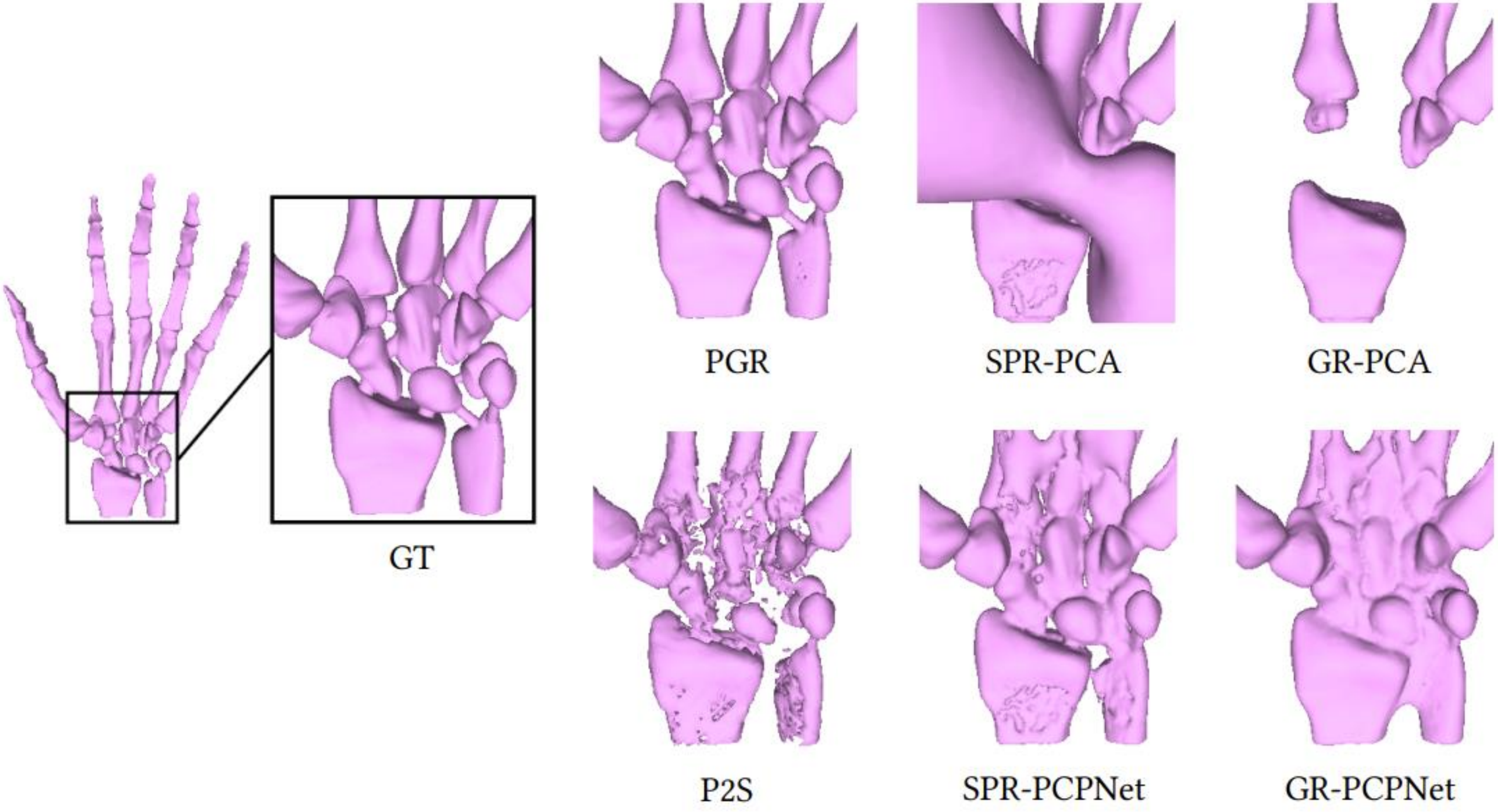
- Geometry converges in **~30 iters**
- Running speed is **~100 iters/sec** on an RTX 3090 with CuPy (40000 points).

3.2 Accuracy

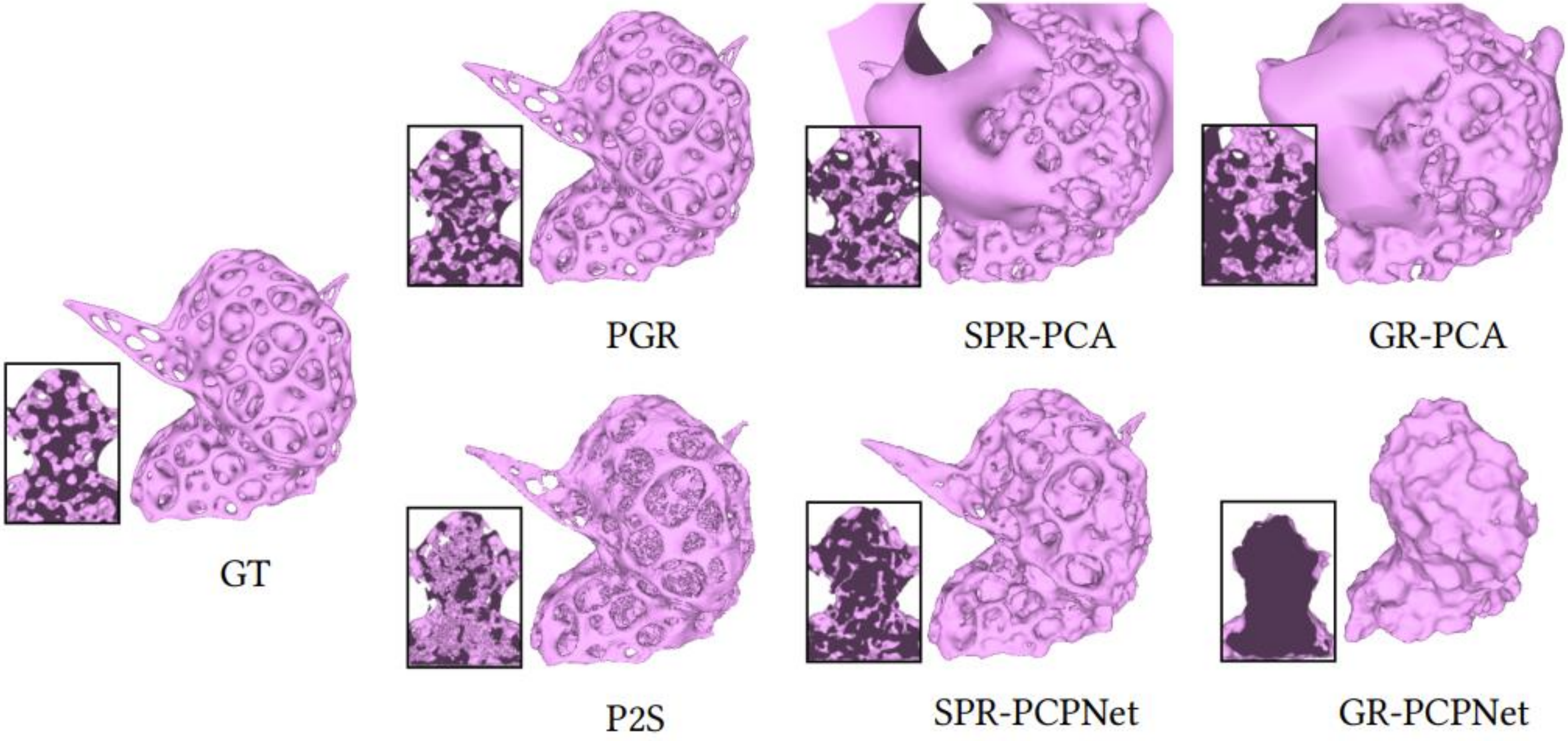
$$(AA^T + \alpha \cdot diag(AA^T))\xi = 1/2, \qquad \xi_{init} = 0$$



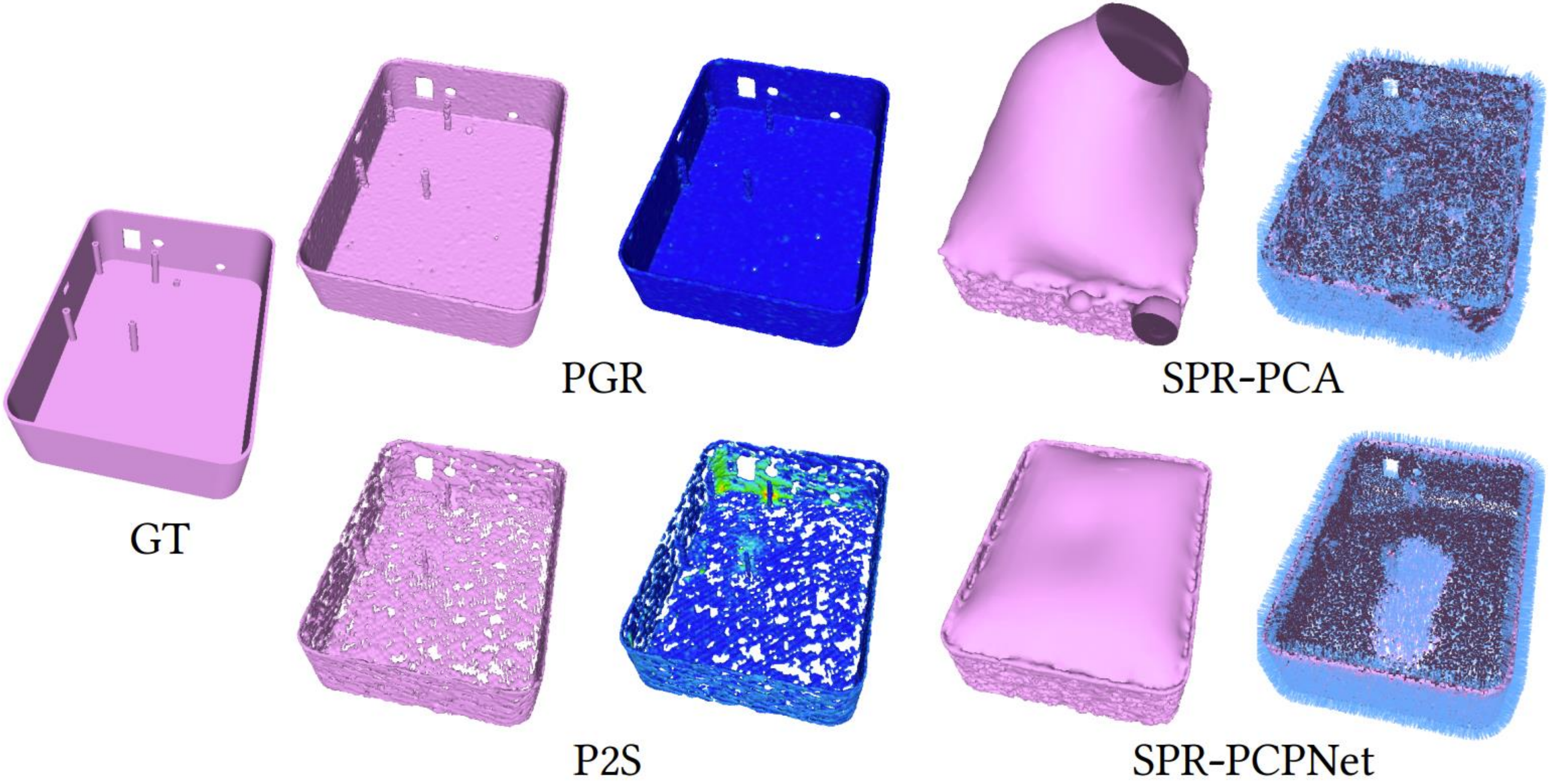
3.3 Difficult Cases: Complex Structure



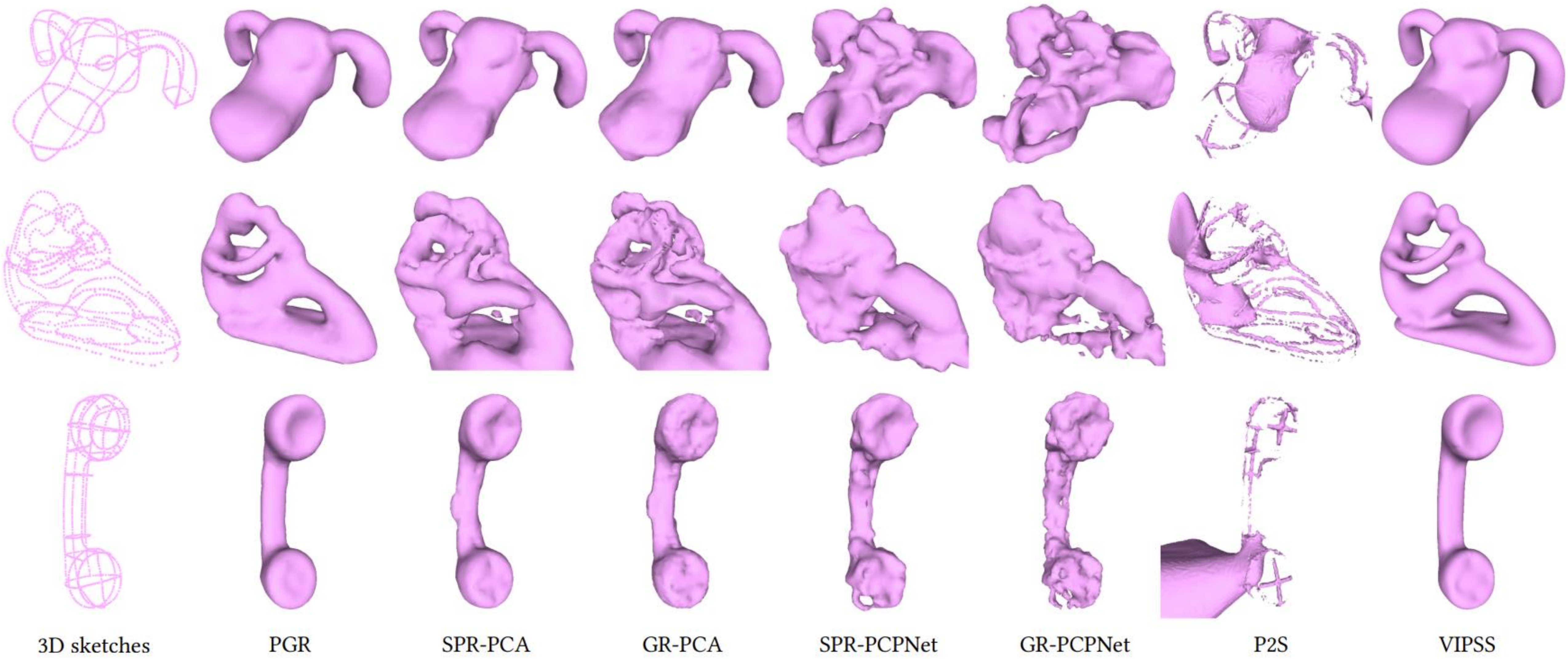
3.3 Difficult Cases: High-genus Surface



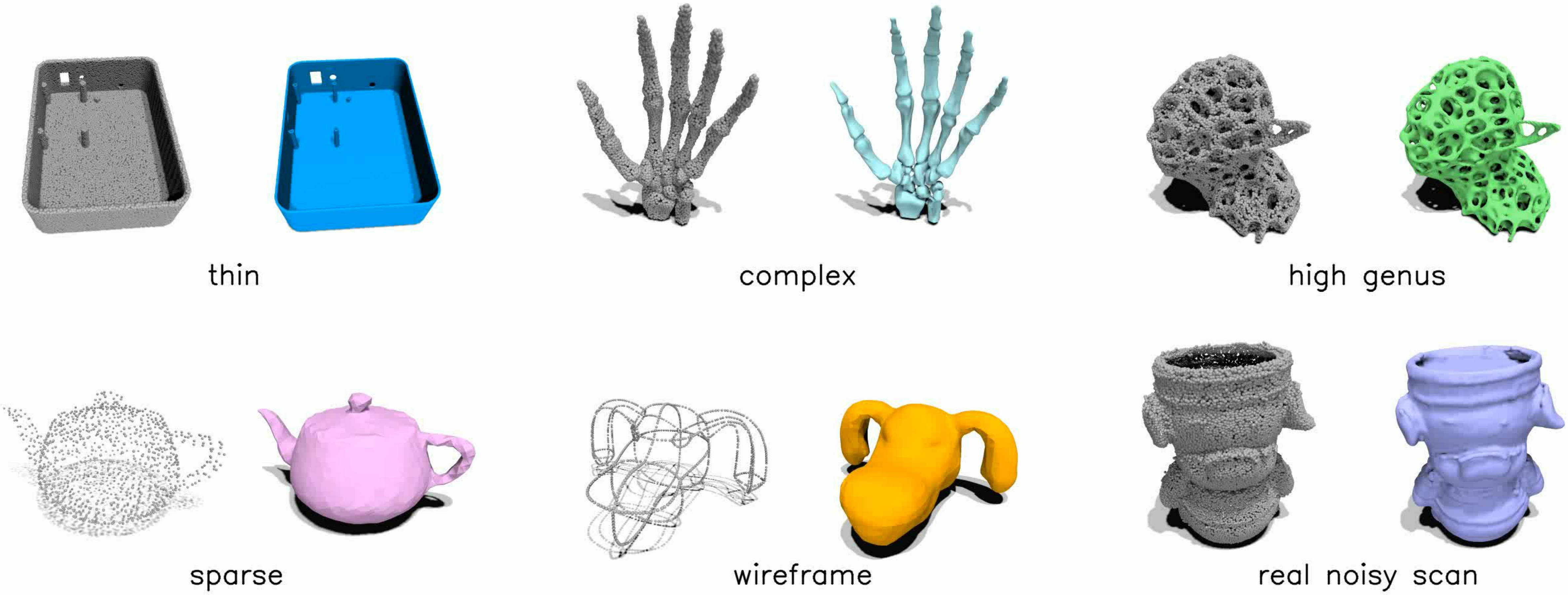
3.3 Difficult Cases: Thin Structure



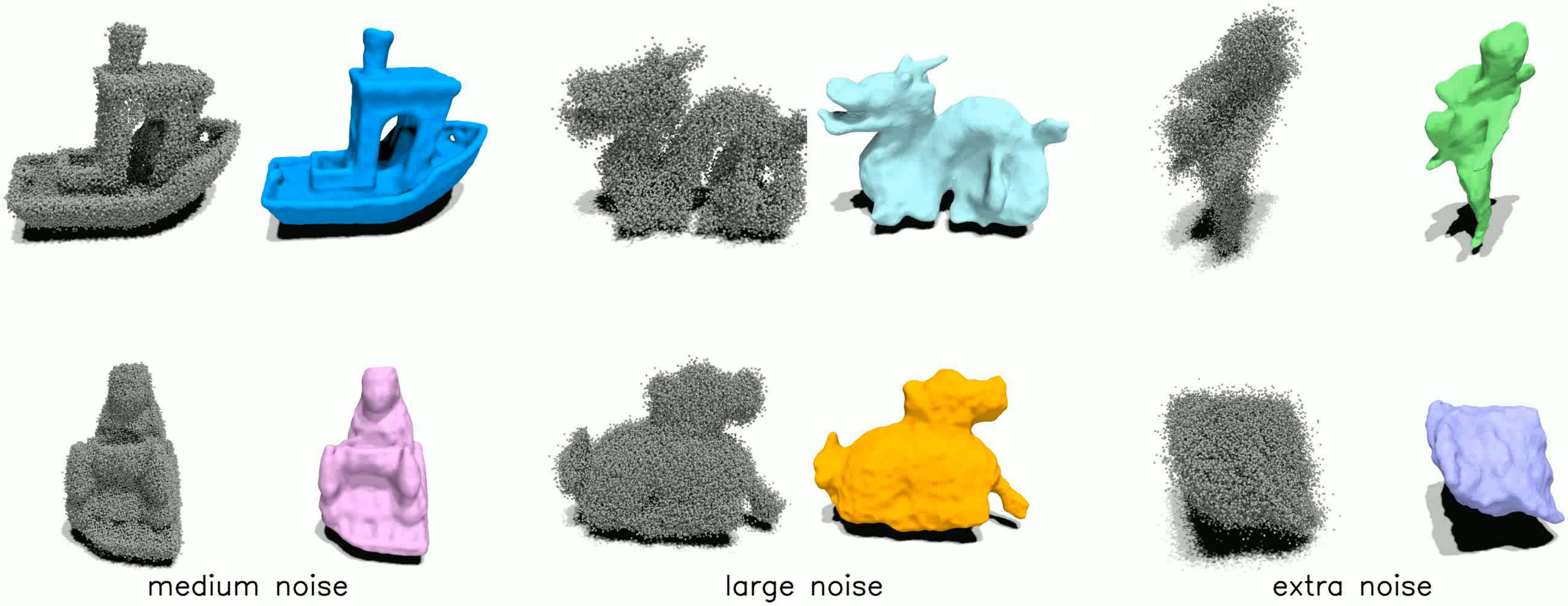
3.3 Difficult Cases: 3D Sketches



3.3 Difficult Cases: 360° Views



3.3 Difficult Cases: Noise Resilience



4 Conclusions & Limitations

- By parametrizing the Gauss formula and viewing normals as parameters, PGR gives a natural representation of the indicator function.

- By using the surface-value constraints in the Gauss formula, PGR can almost always lead to a consistent outward orientation.

4 Conclusions & Limitations

- Efficiency (in seconds):

#points	Build Tree	Precompute AA^T	CG iterations	Field query	MC	Total
10000	0.46	0.98	0.23	0.68	1.02	5.14
40000	1.71	38.61	3.28	7.67	3.90	57.18

  - Time: $O(N^3)$; Storage: $O(N^2)$.

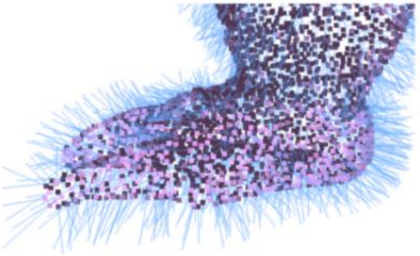
4 Conclusions & Limitations

- Efficiency (in seconds):

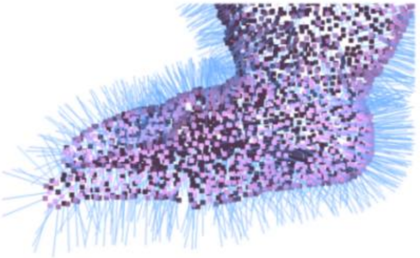
#points	Build Tree	Precompute AA^T	CG iterations	Field query	MC	Total
10000	0.46	0.98	0.23	0.68	1.02	5.14
40000	1.71	38.61	3.28	7.67	3.90	57.18

  - Time: $O(N^3)$; Storage: $O(N^2)$.

- Regularization & Parameter Selection:
  - Our regularization may not be the optimal
  - Optimal parameters are not easy to determine



solved



Accurate normals

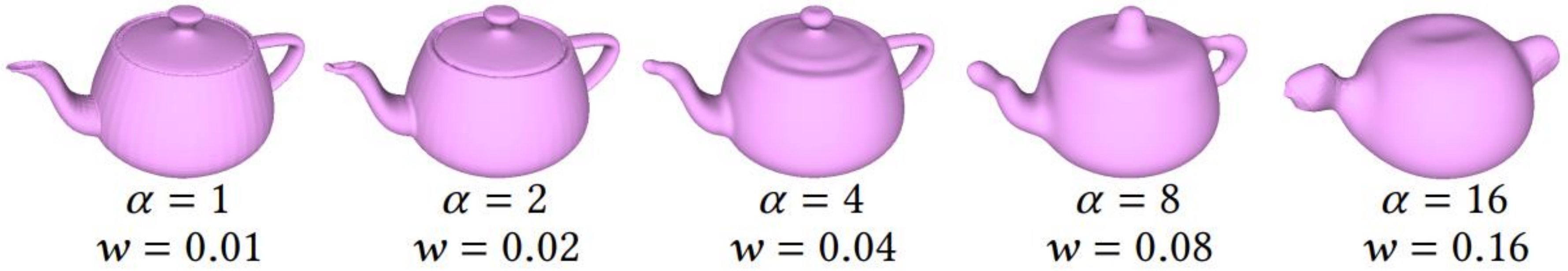
4 Conclusions & Limitations

- Efficiency (in seconds):

#points	Build Tree	Precompute AA^T	CG iterations	Field query	MC	Total
10000	0.46	0.98	0.23	0.68	1.02	5.14
40000	1.71	38.61	3.28	7.67	3.90	57.18

  - Time: $O(N^3)$; Storage: $O(N^2)$.

- Regularization & Parameter Selection:
  - Our regularization may not be the optimal
  - Optimal parameters are not easy to determine



Parametric Gauss Reconstruction

Code is publicly available at
https://github.com/jsnln/ParametricGaussRecon